STAT 2593

Lecture 030 - The One Sample t Test

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The One Sample t Test

Learning Objectives

1. Understand how we test population means in **large sample** non-normally distributed populations or normally distributed populations with unknown variance.

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- Finding the p-value is equivalent to the $N(0, 1)$ case, substituting the normal distribution for a t distribution.

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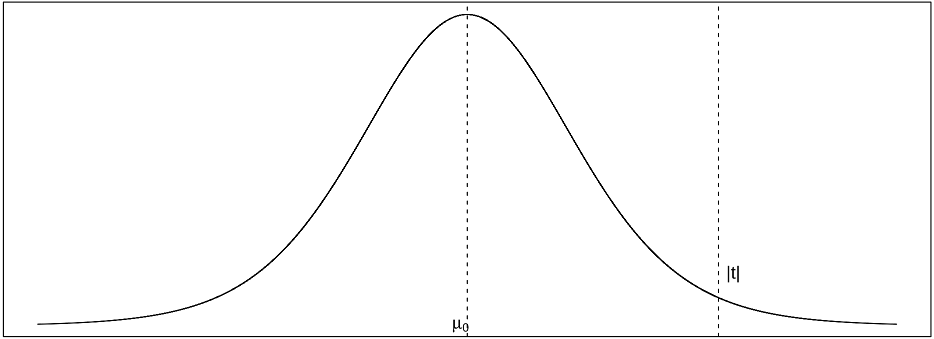

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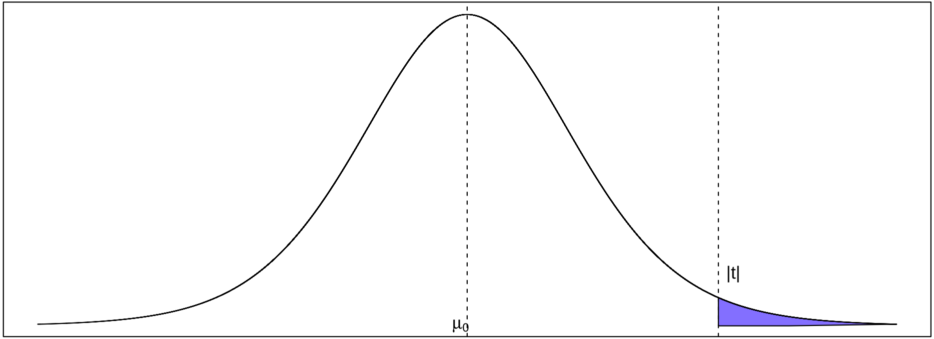

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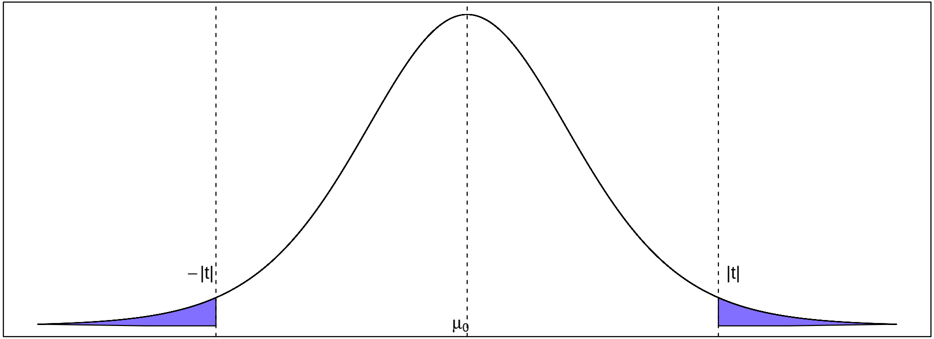

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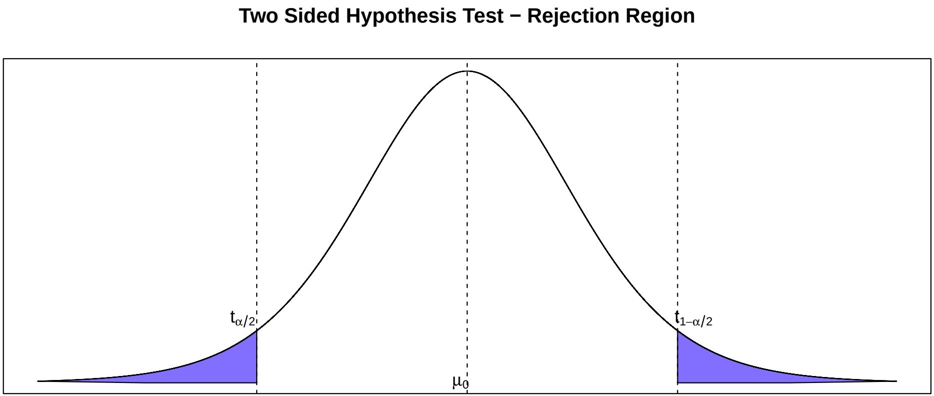
  - Note here we do not take the absolute value.

Rejection Regions for Hypothesis Tests - Critical Values



Summary

- If the sampling distribution is normally distributed, can use a t_{n-1} to run hypothesis tests.

- The rejection region depends on the alternative being considered.